

NOTATION

d , bead diameter; V , v , flow velocity; v_{\max} and v_{\min} , maximum and minimum velocity values, respectively; v_x , v_y , v_τ , and v_n , horizontal, vertical, tangential, and normal components of the velocity, respectively; x and y , coordinates; ρ , liquid density; ϵ , bed porosity; ν , viscosity; p , hydrodynamic pressure; α , resistance coefficient; p_g , pressure in the deformable granular medium; μ , constant; u_i , displacements; γ_{ij} , deformation; Ψ , ψ , stream function; ω , vorticity; H , bed height; h , half width of bed; λ and k , parameters.

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MODEL FOR CALCULATING THE ROTATIONAL FLOW PARAMETERS OF A TWO-PHASE MEDIUM WITH ALLOWANCE FOR PHASE INTERACTION

É. F. Shurgal'skii

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A method is proposed for solving the problem of the rotational flow parameters of a two-phase dusty-gas medium in a cylindrical channel. The effect of the solid particles on the carrier flow is demonstrated numerically.

Apparatus using the cyclone effect make it possible to intensify considerably and qualitatively improve such processes as heat and mass transfer, separation, mixing, and dust collecting. In order to design this apparatus and calculate the operating regimes it is necessary to have data on the hydrodynamic flow parameters of the two-phase medium.

In apparatus with swirling flows the centrifugal forces affect not only the distribution of solid-phase concentration over the cross section [1] but also, what is more important, the aerodynamic characteristics of the carrier gas.

Below we consider a mathematical model and methods of solution applicable to apparatus intended for processing dusty gases. As an example we will take a dust catcher with swirling counterflows (Fig. 1).

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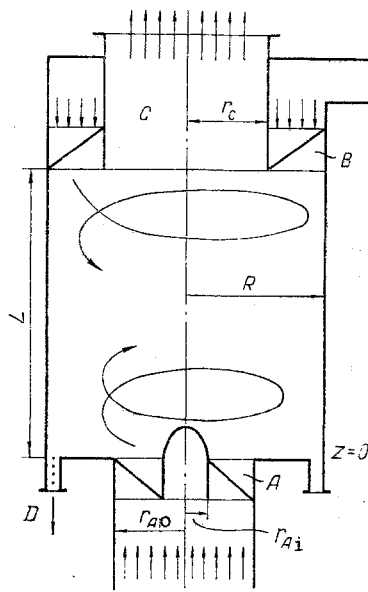


Fig. 1. Flow diagram: A, B) annular swirlers for primary and secondary flows; C) outlet channel; D) annular solid-phase output channel; r_{Ai} , r_{Ao}) inside and outside swirler radii; r_C) outlet channel radius.

If the secondary gas flow through swirler B is single-phase, then in the hydrodynamic parameter calculation scheme its effect can be taken into account as a factor representing injection at given velocities into the primary flow. A two-phase secondary flow considerably complicates the model owing to the need to take the effect of the "third phase" into account.

Let us consider the flow of a two-phase medium consisting of a compressible carrier gas and monodisperse spherical particles in a cylindrical channel of radius R (Fig. 1). We take the axis of symmetry as the Oz axis of the cylindrical coordinates system (z, r, φ) ; the problem is two-dimensional — all the parameters depend on the coordinates z and r , and also on time t . Beyond swirler A a swirling two-phase flow enters the annular orifice $r_{Ai} \leq r < r_{Ao}$ in the initial section of the cylinder $z = 0$; the mean phase velocities have nonzero components in the tangential direction and along the z axis.

We will consider a cylindrical channel of finite length ($0 \leq z \leq L$); the outlet channel C at $z = L$ is fully open, and $L > 2R$.

Within the framework of the interpenetrating continua model the system of equations describing the motion of the two-phase medium in the cylindrical coordinate system, with allowance for the two-dimensionality of the flow, takes the form:

$$\begin{aligned} \frac{\partial r \rho_1}{\partial t} + \frac{\partial r \rho_1 u_{1z}}{\partial z} + \frac{\partial r \rho_1 u_{1r}}{\partial r} &= 0, \\ \frac{\partial r \rho_2}{\partial t} + \frac{\partial r \rho_2 u_{2z}}{\partial z} + \frac{\partial r \rho_2 u_{2r}}{\partial r} &= 0, \\ \frac{\partial z \rho_1 u_{1z}}{\partial t} + \frac{\partial r \rho_1 (u_{1z})^2}{\partial z} + \frac{\partial r \rho_1 u_{1z} u_{1r}}{\partial r} &= -r \frac{\partial p}{\partial z} - r F_z, \\ \frac{\partial r \rho_2 u_{2z}}{\partial t} + \frac{\partial r \rho_2 (u_{2z})^2}{\partial z} + \frac{\partial r \rho_2 u_{2z} u_{2r}}{\partial r} &= r F_z, \\ \frac{\partial r \rho_1 u_{1\varphi}}{\partial t} + \frac{\partial r \rho_1 u_{1\varphi} u_{1z}}{\partial z} + \frac{\partial r \rho_1 u_{1\varphi} u_{1r}}{\partial r} &= -\frac{2 \rho_1 u_{1r} u_{1\varphi}}{r} - F_\varphi, \\ \frac{\partial r \rho_2 u_{2\varphi}}{\partial t} + \frac{\partial r \rho_2 u_{2\varphi} u_{2z}}{\partial z} + \frac{\partial r \rho_2 u_{2\varphi} u_{2r}}{\partial r} &= -\frac{2 \rho_2 u_{2r} u_{2\varphi}}{r} + F_\varphi, \\ \frac{\partial r \rho_1 u_{1r}}{\partial t} + \frac{\partial r \rho_1 (u_{1r})^2}{\partial r} + \frac{\partial r \rho_1 u_{1r} u_{1z}}{\partial z} &= \rho_1 (u_{1\varphi})^2 - r \frac{\partial p}{\partial r} - r F_r, \\ \frac{\partial r \rho_2 u_{2r}}{\partial t} + \frac{\partial r \rho_2 u_{2r} u_{2z}}{\partial z} + \frac{\partial r \rho_2 (u_{2r})^2}{\partial r} &= \rho_2 (u_{2\varphi})^2 + r F_r, \end{aligned}$$

$$\frac{\partial r(\rho_1 E_1 + \rho_2 E_2)}{\partial t} + \frac{\partial r(\rho_1 E_1 u_{1z} + \rho_2 E_2 u_{2z})}{\partial z} + \frac{\partial r(\rho_1 E_1 u_{1r} + \rho_2 E_2 u_{2r})}{\partial r} + \frac{\partial r p u_{1z}}{\partial z} + \frac{\partial r p u_{1r}}{\partial r} = 0,$$

$$\frac{\partial r \rho_2 e_2}{\partial t} + \frac{\partial r \rho_2 e_2 u_{2z}}{\partial z} + \frac{\partial r \rho_2 e_2 u_{2r}}{\partial r} = r q. \quad (1)$$

We supplement system (1) with the equations of state of the phases, assuming that the carrier phase is a perfect gas and the solid particles are incompressible:

$$p = \rho_1^0 (\gamma - 1) e_1, \quad e_1 = c_1 T_1, \quad e_2 = c_2 T_2, \quad \rho_2^0 = \text{const}. \quad (2)$$

The subscripts $i = 1, 2$ relate to the gas and dispersed phases respectively.

The expression for the total energy E_i is written as follows:

$$E_i = e_i + 0,5(u_{iz}^2 + u_{ir}^2 + u_{i\phi}^2). \quad (3)$$

The interphase interaction due to the carrier phase viscosity is determined from the known expression [2, 3]

$$F = 0,75 \frac{C_d \rho_1^0 \rho_2 (\vec{u}_1 - \vec{u}_2) |\vec{u}_1 - \vec{u}_2|}{\rho_2^0 d}. \quad (4)$$

The drag coefficient C_d is calculated from the empirical formulas for specific intervals of Reynolds numbers based on the diameter of the spherical particle:

$$C_d = 24 \text{Re}^{-1} + 4 \text{Re}^{-0,33} \quad \text{for } 0 < \text{Re} \leq 700,$$

$$C_d = 4,3 (\lg \text{Re})^{-2} \quad \text{for } 700 < \text{Re} \leq 2000,$$

$$\text{Re} = \rho_1^0 |\vec{u}_1 - \vec{u}_2| d / \mu.$$

The interphase heat transfer entering into system (1) is assumed to be proportional to the phase temperature difference [2]:

$$q = \pi n d \text{Nu} \lambda (T_1 - T_2). \quad (5)$$

Here n is the number of solid particles per unit volume; $\text{Nu} = 2 + 0,6 \text{Re}^{1/2} \text{Pr}^{1/3}$. In its turn, the Prandtl number $\text{Pr} = C_p \mu / \lambda$.

From system (1) it is clear that the viscosity and thermal conductivity of the carrier phase are taken into account only in the terms describing the interphase momentum and energy transfer; the buoyant force, proportional to the particle fraction, is not taken into account, nor are the forces associated with the apparent mass and the rotation of the particles.

The following boundary conditions apply: (1) for the particles the no-reflection condition is imposed at the walls; this means that particles striking the channel walls disappear from the flow; (2) there is no leakage of carrier gas through the cylinder walls (no flow conditions); (3) at the channel inlet the velocities of both phases, the pressure and density of the gas, and the density of the solid phase are all specified as follows:

$$z = 0, \quad r_{Ai} \leq r \leq r_{Ao}, \quad \vec{u}_1 = \vec{u}_{10}, \quad u_2 = u_{20}, \quad p = p_0, \quad \rho_1 = \rho_{10}, \quad \rho_2 = \rho_{20} \quad (6)$$

(it should be noted that \vec{u}_{10} and \vec{u}_{20} may be different); (4) at the outlet from the cylinder in the section $z = L$ (fully open channel) we require that the flow be homogeneous along the z axis and hence that the derivatives of the flow parameters with respect to the axial coordinate be zero; (5) at $t = 0$ we have a uniform equilibrium distribution of the parameters of the two-phase medium in the channel:

$$\begin{aligned} u_{1z}(z, r, 0) &= u_{2z}(z, r, 0) = u_{z0}, \\ u_{1r}(z, r, 0) &= u_{2r}(z, r, 0) = u_{1\phi}(z, r, 0) = u_{2\phi}(z, r, 0) = 0, \\ T_1(z, r, 0) &= T_2(z, r, 0) = T_0, \\ \rho_1(z, r, 0) &= \rho_{10}, \quad \rho_2(z, r, 0) = \rho_{20}. \end{aligned} \quad (7)$$

The stationary solution of this problem is found numerically, and in the process of calculating variants with different boundary values (6), for convenience and to save computation time, the steady-state parameter distributions of the previous variant were sometimes employed as initial conditions (7).

Equations (1), (2), (6), and (7) were solved numerically by the method of large particles [4]. This method has been successfully employed for calculating supersonic two-phase flows; for subsonic velocities a modification of the method has been proposed in which for the Euler stage a difference scheme implicit with respect to time is used for calculating the pressure [5].

Meanwhile, in the case of the problem in question, at sufficiently small subsonic velocities ($M \leq 0.1$), calculations in accordance with this method have shown that it is not very accurate. Thus, when $M = 0.1$ the difference between the pressure and the density of the carrier phase in the stationary solution and the initial distributions (7) was 10-15%, although simple estimates using the Bernoulli integral show that this difference should be within 1.5-2%. In this paper, in order to improve the accuracy of the method of large particles in the case of small subsonic velocities another algorithm for calculating the Euler stage is proposed.

For this purpose let us consider the equations describing the motion of the carrier phase in the Euler stage in dimensionless variables (all the parameters are divided by the corresponding values at the channel inlet, and the variables r, z, t by the channel radius R and the characteristic time R/u_{1z0} , where u_{1z0} is the given value of the axial gas velocity component at the inlet):

$$\rho \frac{\partial u}{\partial t} = -\frac{1}{\varepsilon} \frac{\partial p}{\partial z}, \quad \rho \frac{\partial v}{\partial t} = -\frac{1}{\varepsilon} \frac{\partial p}{\partial r}, \quad (8)$$

$$\frac{\partial p}{\partial r} + (\gamma - 1) p \operatorname{div} \vec{V} = 0.$$

In (8) $\varepsilon = \gamma M^2$; u, v , and ρ are the dimensionless axial and radial components of the velocity vector \vec{V} and the density of the carrier phase. We write Eqs. (8) in difference form in accordance with the implicit scheme:

$$u_{i,j}^{n+1} = u_{i,j}^n - \frac{\kappa}{\varepsilon \rho_{i,j}^n} (p_{i+1/2,j}^{n+1} - p_{i-1/2,j}^{n+1}), \quad (9)$$

$$v_{i,j}^{n+1} = v_{i,j}^n - \frac{\kappa}{\varepsilon \rho_{i,j}^n} (p_{i,j+1/2}^{n+1} - p_{i,j-1/2}^{n+1}),$$

$$p_{i,j}^{n+1} = p_{i,j}^n - \kappa(\gamma - 1) p_{i,j}^n [(u_{i+1/2,j}^{n+1} - u_{i-1/2,j}^{n+1}) + (v_{i,j+1/2}^{n+1} - v_{i,j-1/2}^{n+1})],$$

$$\kappa = \tau/h. \quad (10)$$

Here τ and h are the time and coordinate steps; the superscripts relate to the time step, the subscripts to the number of the cell; the fractional subscripts correspond to the half-sum of the values of the parameter in neighboring cells [4, 5].

We eliminate from equation (9) the values of the pressure $p_{j,i}^{n+1}$ in time step $n+1$ by means of equation (10). Introducing the notation

$$A_i = \frac{\kappa^2(\gamma - 1) p_{i-1/2,j}^n}{\varepsilon \rho_{i,j}^n}, \quad B_i = \frac{\kappa^2(\gamma - 1) p_{i+1/2,j}^n}{\varepsilon \rho_{i,j}^n}, \quad C_i = 1 + A_i + B_i,$$

$$F_i = -u_{i,j}^n + \frac{\kappa}{\varepsilon \rho_{i,j}^n} (p_{i+1/2,j}^n - p_{i-1/2,j}^n) - \frac{\kappa^2(\gamma - 1)}{\varepsilon \rho_{i,j}^n} \times$$

$$\times [p_{i+1/2,j}^n (v_{i+1/2,j+1/2}^{n+1} - v_{i+1/2,j-1/2}^{n+1}) - p_{i-1/2,j}^n (v_{i-1/2,j+1/2}^{n+1} - v_{i-1/2,j-1/2}^{n+1})],$$

$$A_j^* = \frac{\kappa^2(\gamma - 1) p_{i,j-1/2}^n}{\varepsilon \rho_{i,j}^n}, \quad B_j^* = \frac{\kappa^2(\gamma - 1) p_{i,j+1/2}^n}{\varepsilon \rho_{i,j}^n}, \quad C_j^* = 1 + A_j^* + B_j^*$$

$$F_j^* = -v_{i,j}^n + \frac{\kappa}{\varepsilon \rho_{i,j}^n} [p_{i,j+1/2}^n - p_{i,j-1/2}^n] - \frac{\kappa^2(\gamma - 1)}{\varepsilon \rho_{i,j}^n} \times$$

$$\times [p_{i,j+1/2}^n (u_{i+1/2,j+1/2}^{n+1} - u_{i-1/2,j+1/2}^{n+1}) - p_{i,j-1/2}^n (u_{i+1/2,j-1/2}^{n+1} - u_{i-1/2,j-1/2}^{n+1})].$$

we obtain a system of equations for $u_{i,j}^{n+1}$ and $v_{i,j}^{n+1}$, which does not contain the pressure $p_{i,j}^{n+1}$, in the form

$$A_i u_{i-1,j}^{n+1} - C_i u_{i,j}^{n+1} + B_i u_{i+1,j}^{n+1} = F_i, \quad (11)$$

$$A_j^* v_{i,j-1}^{n+1} - C_j^* v_{i,j}^{n+1} + B_j^* v_{i,j+1}^{n+1} = F_j^*. \quad (12)$$

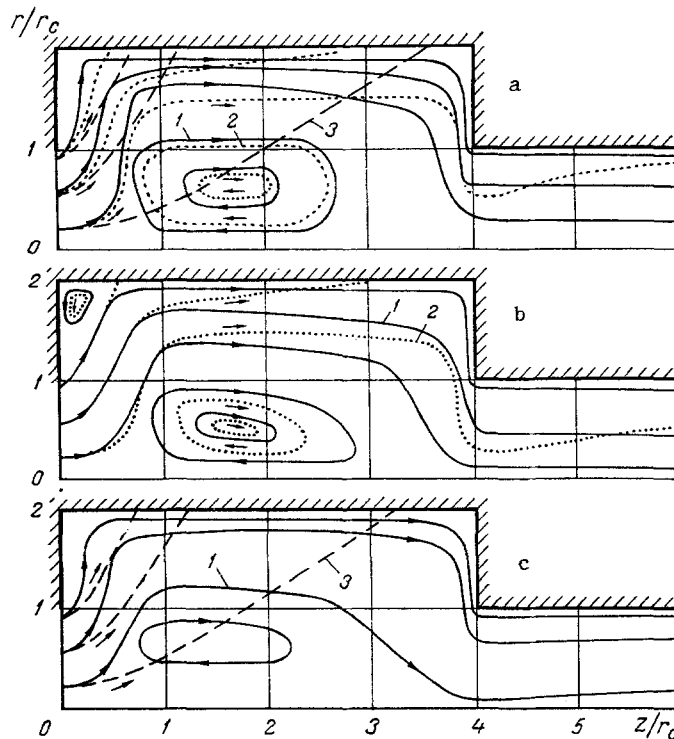


Fig. 2. Projections of streamlines of carrier phase (1) and solid particles 8 and 80 μm in size (2, 3) at mass concentrations: a) $\rho_{20}/\rho_{10} = 0.004$; b, c) $\rho_{20}/\rho_{21} = 1$.

Linear system of equations (11), (12) was solved by the longitudinal-transverse pivotal method using iterations with respect to u and v , by means of which system (11) was decoupled from system (12). The values of $u_{i,j}^{n+1}$ and $v_{i,j}^{n+1}$ entering into the expressions for F_i and F_j^* were replaced by the corresponding values taken from the previous iteration. After system (11), (12) has been solved with given accuracy, the Euler stage distribution is determined from equations (11). The remaining stages of the calculation are completed in the same way as in [4, 5].

Calculations carried out using this algorithm showed that in the steady-state solution for Mach numbers of the order of 0.1 the difference between the density and the pressure of the carrier phase within the channel and their values at the inlet is less than 1%; in this case in the calculations we used the rather large $\kappa = 0.1$.

In order to estimate the effect of the solid particles on the hydrodynamics of the carrier phase, using a BÉSM computer and the method developed above we carried out a numerical experiment to determine the flow parameters for mass concentrations $\rho_{20}/\rho_{10} = 0.004$ and 1 and particle sizes $d = 8$ and $80 \mu\text{m}$ at: (1) a carrier flow Mach number $M = 0.08$; (2) a ratio of the tangential and axial velocity components $u_{1\varphi 0}/u_{1z 0} = 3.6$; (3) a ratio of the total length of the cylinder L to its radius R $L/R = 6$; (4) a flow inlet section $r_{A0}/R = 1/2$, $r_{A1} = 0$ (Fig. 1); (5) the structure of the apparatus and the secondary-flow inlet conditions can be simulated by introducing steps, as shown in Fig. 2, where the shape and size of the step are characterized by the radius $r_C = R/2$ on the section $4 \leq z/L \leq 6$.

In Fig. 2 the projections of the streamlines of the gas and the solid particles on the longitudinal section are represented in r/r_C and z/r_C coordinates, where r_C is the radius of the outlet channel C (Fig. 1).

The data of the numerical experiment presented in Fig. 2a show that at low mass concentrations $\rho_{20}/\rho_{10} = 0.004$ the projections of the carrier phase streamlines (continuous curves 1) for particles 8 and $80 \mu\text{m}$ in size coincide, i.e., the aerodynamics of the carrier phase are unaffected by the solid particles.

The data of Fig. 2a show that the greatest differences between the streamlines of the gas (curves 1) and solid phases in the case of small particles ($8 \mu\text{m}$) (curves 2) are observed in the peripheral parts of the channel. In the axial region the trajectories of the solid and gas phases almost coincide. When the particle size is increased by an order, the differ-

ences between the streamlines of the gas (curves 1) and solid (curves 3) phases becomes considerable, the form of the solid-phase streamlines being unaffected by the mass concentration on the interval $\rho_{20}/\rho_{10} = 0.004-1$ (Figs. 2a and 2c).

From Fig. 2c it also follows that an increase in particle size results in a decrease in the size of the region of existence of the toroidal eddy, whereas the projections of the carrier-phase streamlines in the wall region are similar in nature to those of Fig. 2a. It is also clear that at a distance of the order of only two diameters the flow is already free of solid particles. Increasing the size of the particles leads to a shortening of the path on which phase separation takes place.

On comparing Figs. 2b and 2c we found that increasing the mass concentration ($\rho_{20}/\rho_{10} = 1$) modifies both the form of the projection of the carrier-phase streamlines and their position in the coordinate grid. An increase in the mass concentration of the 8- μm particles "smooths" the streamlines in the axial direction at the same time as deforming the region of the toroidal eddy.

Thus, from Figs. 2b and 2c we may conclude that the nature of the motion of the carrier phase is affected by both the absolute magnitude of the mass concentration and the size of the solid particles contained in the flow.

Numerical experiments carried out, for example, to investigate the configuration of the inlet system, the method of swirling the incoming flow, the reverse flows, and the conditions of separation of the solid phase and to determine the profile of the velocity components, together with control comparisons of the theoretical and experimental data, indicate that the proposed method of determining the parameters of the two-phase flow makes it possible to perform a detailed functional analysis and to solve problems of the optimal design of apparatus for treating dusty gases.

NOTATION

ρ_i , mean density of the i -th phase; ρ_i^0 , true density of the i -th phase; c_2 , specific heat of the particles; c_v , specific heat of the gas at constant volume; μ , dynamic viscosity of the gas; λ , thermal conductivity of the gas; c_p , specific heat of the gas at constant pressure; γ , specific heat ratio of the gas; d , particle diameter; R , radius of the cylindrical channel; u_i , velocity vector of the i -th phase; u_{zi} , u_{ir} , $u_{i\phi}$, components of the velocity vector of the i -th phase; u_{i20} , axial component of the gas velocity in the channel inlet section; p , pressure of the gas phase; e_i , internal energy of the i -th phase; E_i , total energy of the i -th phase; T_i , temperature of the i -th phase; C_d , particle drag coefficient; Re , Reynolds number of the relative flow past the particle; Nu , Nusselt number; Pr , Prandtl number; M , Mach number of the carrier flow based on the axial velocity in the inlet section. Indices: i , phase number; 0 , the corresponding parameters are taken in the inlet section; i, j , cell number in the difference equations; n , number of the interior layer in the difference equations.

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